Exponential Functions

What You’ll Learn
• To evaluate exponential functions
• To graph exponential functions

...And Why
To use an exponential model for a population of rabbits, as in Example 2

1 Evaluating Exponential Functions

The rules you wrote in Lesson 8-6 to describe geometric sequences, such as
\( A(n) = 3 \cdot 4^{n-1} \), are examples of exponential functions.

**Key Concepts**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Exponential Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>An exponential function is a function in the form ( y = a \cdot b^x ), where ( a ) is a nonzero constant, ( b ) is greater than 0 and not equal to 1, and ( x ) is a real number.</td>
<td></td>
</tr>
</tbody>
</table>

Examples
- \( y = 0.5 \cdot 2^x \)
- \( f(x) = -2 \cdot 0.5^x \)

You can evaluate an exponential function for given values of the domain to find the corresponding values of the range.

**EXAMPLE**

Evaluate each exponential function.

a. \( y = 5^x \) for \( x = 2, 3, 4 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 5^x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5^2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>5^3</td>
<td>125</td>
</tr>
<tr>
<td>4</td>
<td>5^4</td>
<td>625</td>
</tr>
</tbody>
</table>

b. \( t(n) = 4 \cdot 3^n \) for the domain \([-3, 6]\)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 4 \cdot 3^n )</th>
<th>( t(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4 \cdot 3^{-3} = 4 \cdot \frac{1}{27} = \frac{4}{27}</td>
<td>\frac{4}{27}</td>
</tr>
<tr>
<td>6</td>
<td>4 \cdot 3^6 = 4 \cdot 729 = 2916</td>
<td>2916</td>
</tr>
</tbody>
</table>

Evaluate each exponential function for the domain \([-2, 0, 3]\).

a. \( y = 4^x \) for \( x = \frac{1}{16}, 1, 64 \)

b. \( f(x) = 10 \cdot 5^x \) for \( x = \frac{5}{2}, 10, 1250 \)

c. \( g(x) = -2 \cdot 3^x \) for \( x = -\frac{5}{3}, -2, -54 \)
You can evaluate exponential functions to solve real-world problems.

**EXAMPLE**  
**Real-World Problem Solving**

**Gridded Response** Suppose 20 rabbits are taken to an island. The rabbit population then triples every half year. The function $f(x) = 20 \cdot 3^x$, where $x$ is the number of half-year periods, models this situation. How many rabbits would there be after 2 years?

$$f(x) = 20 \cdot 3^x$$

= $20 \cdot 3^4$

= $20 \cdot 81$

= 1620

Simplify.

In 2 years, there are 4 half years. Evaluate the function for $x = 4$.

After two years, there would be 1620 rabbits.

**2. Teach**

**Guided Instruction**

- **PowerPoint Additional Examples**
  - 1. Evaluate each exponential function.
    - a. $y = 3^x$ for $x = 2, 3, 4, 9, 27, 81$
    - b. $p(q) = 3 \cdot 4^q$ for the domain $(-2, 3)$

- **Quick Check**
  - 2. Suppose two mice live in a barn. If the number of mice quadruples every 3 months, how many mice will be in the barn after 2 years? 131,072

**Additional Examples**

- 3. Graph $y = 2 \cdot 3^x$. See back of book.

- 4. The function $f(x) = 1.25^x$ models the increase in size of an image being copied over and over at 125% on a photocopier. Graph the function. See back of book.

**Resources**

- Daily Notetaking Guide 8-7 L3
- Daily Notetaking Guide 8-7—Adapted Instruction L1

**Closure**

Ask students to explain why $y = x^4$ is not an exponential function. The exponent, not the base, must be a variable. Have students explain how the graphs of $y = x^4$ and $y = 4^x$ differ. The graph of $y = x^4$ is a u-shape in which the $y$-values increase on both sides of the $y$-axis as you move away from the $y$-axis. The graph of $y = 4^x$ is a smooth curve in which the $y$-values increase quickly as you move to the right side of the $y$-axis and decrease slowly as you move to the left of the $y$-axis.

**Advanced Learners**

- Lead students in a discussion of the graph of $y = a^x$, when $a$ is greater than or equal to 1 and when $a$ is between 0 and 1.

**English Language Learners**

- Be sure students understand the term exponential function. Ask them to write a function that describes $y$ as a function of $x$, and then a function for $y$ as an exponential function of $x$. Discuss how the functions differ.

**Learning Styles**

- visual
- verbal
You can graph exponential functions to model real-world situations.

**Example 2 (page 469)**

**Photocopying** Many photocopiers allow you to choose how large you want an image to be. The function $f(x) = 1.5^x$ models the new size of an image being copied over and over at 150%, where $x$ is the number of enlargements. Graph the function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$1.5^x$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>(1, 1.5)</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>(2, 2.25)</td>
</tr>
<tr>
<td>3</td>
<td>3.375</td>
<td>(3, 3.375)</td>
</tr>
<tr>
<td>4</td>
<td>5.0625</td>
<td>(4, 5.0625)</td>
</tr>
<tr>
<td>5</td>
<td>7.59375</td>
<td>(5, 7.59375)</td>
</tr>
</tbody>
</table>

**Critical Thinking** Explain why the function in Example 4 models discrete data. It is not possible to have a fractional number of enlargements.

**EXERCISES**

**Practice and Problem Solving**

Evaluate each function rule for the given value.

1. $f(x) = 6^x$ for $x = 3$ \(\text{216}\)  
2. $g(t) = 2 \cdot 3^t$ for $t = -2$ \(\text{\frac{2}{9}}\)
3. $y = 20 \cdot (0.5)^x$ for $x = 3$ \(\text{2.5}\)
4. $h(w) = 0.5 \cdot 4^w$ for $w = 3$ \(\text{32}\)
5. $y = 50 \cdot (0.3)^x$ for $x = 2$ \(\text{4.5}\)
6. $f(x) = 1.8 \cdot 2^x$ for $x = 6$ \(\text{115.2}\)
7. $y = 100 \cdot \left(\frac{1}{2}\right)^x$ for $x = -4$ \(\text{1600}\)
8. $y = 9 \cdot \left(\frac{2}{3}\right)^x$ for $x = -3$ \(\text{0.576}\)

**Example 2 (page 469)**

**Finance** Suppose an investment of $10,000 doubles in value every $13$ years. How much is the investment worth after $52$ years? After $65$ years? \(\text{See left.}\)

**Finance** Suppose an investment of $500$ doubles in value every $15$ years. How much is the investment worth after $30$ years? After $45$ years? \(\text{See left.}\)

**Finance** Suppose an investment of $2000$ doubles in value every $8$ years. How much is the investment worth after $24$ years? After $32$ years? \(\text{$16,000$, $32,000$}\)

**Example 3 (page 469)**

Match each table with the function that models the data.

**Example 3 (page 469)**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

470 Chapter 8 Exponents and Exponential Functions
Exponential Functions

Suppose you are photocopying an image, reducing it to 85% its original size. The function \( f(x) = 0.85^x \) models the size of an image after \( x \) times it is reduced. Graph the function. See margin.

A population of 100 insects triples in size every month. The function \( y = 100 \cdot 3^x \) models the population after \( x \) months. Graph the function.

23. Photocopying Suppose you are photocopying an image, reducing it to 85% its original size. The function \( y = 0.85^x \) models the size of an image after \( x \) number of times it is reduced. Graph the function. See margin.

24. Science A population of 100 insects triples in size every month. The function \( y = 100 \cdot 3^x \) models the population after \( x \) months. Graph the function. See left above.

Evaluate each function for the domain \( \{-2, -1, 0, 1, 2, 3\} \). As the values of the domain increase, do the values of the range increase or decrease? 25–32. See left.

25. \( f(x) = 5^x \) 26. \( y = 2.5^x \) 27. \( h(x) = 0.1^x \) 28. \( f(x) = 5 \cdot 4^x \) 29. \( y = 0.5^x \) 30. \( y = \left(\frac{2}{3}\right)^x \) 31. \( g(x) = 4 \cdot 10^x \) 32. \( y = 100 \cdot 0.3^x \)

33. Multiple Choice The population of Texas in 2000 was about 20,852 million people. The function \( p(n) = 20.852(1.02071)^n \) estimates the population where \( n = 0 \) corresponds to the year 2000. Which is a reasonable estimate in millions of the population of Texas in 2020? B

21.284  31.420  41.740  49.842

34. Biology A certain species of bacteria in a laboratory culture begins with 75 cells and doubles in number every 20 min.

a. Copy, complete, and extend the table to find when there will be more than 5,000 bacteria cells. See back of book.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Number of 20-min Time Periods</th>
<th>Pattern</th>
<th>Number of Bacteria Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>75 \cdot 2</td>
<td>75 \cdot 2 = II</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
<td>75 \cdot 2 \cdot 2</td>
<td>75 \cdot 2^2 = II</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>75 \cdot 2 \cdot 2 \cdot 2</td>
<td>75 \cdot 2^3 = II</td>
</tr>
</tbody>
</table>

b. Write a function rule to model the situation. \( y = 75 \cdot 2^x \), where \( x \) is the number of 20-min time periods.

Lesson 8-7  Exponential Functions
1. Evaluate each function rule for the given value.
   a. \( y = 0.5^x \) for \( x = 3 \) \( \frac{1}{2} \)
   b. \( f(x) = 4 \cdot 3^x \) for \( x = -2 \) \( \frac{1}{3} \)
2. Suppose an investment of $5000 doubles every 12 years.
   a. How much is the investment worth after 24 years? $20,000
   b. After 48 years? $80,000
3. Graph \( y = 0.5 \cdot 3^x \).
4. Graph \( y = -0.5 \cdot 3^x \).

**Alternative Assessment**

Write \( y = 3x \) on a transparency and project it with an overhead projector. Give students three seconds to look at the equation and write on their own paper whether the equation is exponential or not exponential. Repeat with various functions. Cover each function as you proceed. Include exponential, linear, quadratic, and absolute value functions. At the end of the activity, uncover the whole list of functions. Have students compare their answers with those of classmates and determine which are correct.

**Test Prep**

**Multiple Choice**

51. For the function \( y = -3^x \), what is the value of \( y \) when \( x = -2 \)?
   A. \(-9\)  B. \(-\frac{1}{9}\)  C. \(\frac{1}{9}\)  D. 9

52. Which function contains the points \((1, 3)\) and \((3, 6.75)\)?
   G. \( f(x) = 1.675x + 1.325 \)
   H. \( f(x) = 1.5 \cdot 2^x \)
   J. \( f(x) = 1.325x + 1.675 \)

53. Which function has the same \( y \)-intercept as \( y = 2^x \)?
   A. \( y = x + 1 \)
   B. \( y = 2x \)
   C. \( y = x \)
   D. \( y = 2(x + 1) \)

**Exercises**

35a. Graph \( y = 2^x \) and \( y = 4^x \).

b. Between \( x = 1 \) and \( x = 3 \), the graph of \( y = x^2 \) rises faster than the graph of \( y = 2^x \). The graphs intersect at \( x = 2 \).
54. A population of 6000 doubles in size every 10 years. Which equation relates the size of the population \( y \) to the number of 10-year periods \( x \)?

\[
\begin{align*}
F. \quad y &= 6000 \cdot 10^x \\
G. \quad y &= 10 \cdot 2^x \\
H. \quad y &= 6000 \cdot 2^x \\
J. \quad y &= 2 \cdot 100^x
\end{align*}
\]

55. Between what two integer values of \( x \) do the graphs of \( y = 20(0.5)^x \) and \( y = 0.5 \cdot 4^x \) intersect? Show your work. *See margin.*

56. Write a rule for the sequence.

57. Find each common ratio. Then find the next three terms in each sequence.

58. 2, 10, 50, 250, ____, ____, ____.

59. 3, 12, 48, ____, ____, ____.

60. 450, 45, 4.5, 0.45, ____, ____, ____.

61. 7168, 1792, 448, 112, ____, ____, ____.

62. Write a line that passes through the given point and is parallel to the given line.

63. Find each common ratio. Then find the next three terms in each sequence.

64. Write an equation for the line that passes through the given point and is parallel to the given line.

65. Write an equation for the line that passes through the given point and is parallel to the given line.

### Checkpoint Quiz 2

#### Lessons 8-5 through 8-7

1. Simplify each expression.

   \[ \left( \frac{2^2}{3-1} \right)^4 \quad 3^{12} \]

   \[ 2. \left( \frac{2^2}{7} \right)^5 \quad \frac{y^{16}}{y^{20}} \]

   \[ 3. \left( \frac{10m^{-3}}{25n^6} \right)^2 \quad \frac{4n^{12}}{25m^{10}} \]

   \[ 4. \left( \frac{2^{1/3}}{6^{1/2}} \right)^2 \quad \frac{1}{x^{10}} \]

2. Determine whether each sequence is arithmetic or geometric.

   5. 22, 55, 1375, ____, ____, ____.

   6. 5, 10, 20, 40, 80, ____, ____, ____.

   7. 5, 10, 15, 20, 25, ____, ____, ____.

   8. Use the sequence \(-100, 20, -4, \ldots\)

   a. What is the first term? \(-100\)

   b. What is the common ratio? \(-\frac{1}{5}\) or \(-0.2\)

   c. Write a rule for the sequence. \(a_n = -100 \cdot (-0.2)^{n-1}\)

   d. Use your rule to find the fifth and seventh terms in the sequence.

   \(-0.16\); \(-0.0064\)

9. **Physics** On the first swing, a pendulum swings through an arc of length 40 cm. On each successive swing, the length of the arc is 85% of the length of the previous swing.

   a. Write a rule to model this situation. \(a_n = 40 \cdot (0.85)^{n-1}\)

   b. Find the length of the arc on the fifth swing. Round your answer to the nearest millimeter. **209 mm**

10. **Commuting** Refer to the information at the left.

    a. Write the number of vehicles that crossed the George Washington Bridge in scientific notation. \(1.08 \times 10^8\)

    b. The Port Authority collected about $249 million in tolls from this bridge. Write this number in scientific notation. \(2.49 \times 10^8\)

    c. What was the average toll per vehicle? **about $2.31**

### Mixed Review

**Lesson 8-6**

Find each common ratio. Then find the next three terms in each sequence.

66. 2, 10, 50, ____, ____, ____.

67. 7, -21, 63, ____, ____, ____.

68. -0.2, -0.4, -0.8, ____, ____, ____.

69. 27, -9, 3, ____, ____, ____.

70. 450, 45, 4.5, 0.45, ____, ____, ____.

71. 7168, 1792, 448, 112, ____, ____, ____.

72. 0.1; 0.045, 0.0045, 0.00045.

Write an equation for the line that passes through the given point and is parallel to the given line.

73. \(y = 5x + 1; \) \((0, 0) \) \(y = 5x\)

74. \(y = 3x - 2; \) \((0, 1) \) \(y = 3x + 1\)

75. \(y = -2x + 5; \) \((4, 0) \) \(y = -2x + 8\)

76. \(y = 0.4x + 5; \) \((2, -3) \) \(y = 0.4x - 3.8\)

### Online Lesson Quiz

PHSchool.com, Web Code: ata-0807